August 2000

Next-to-next-to-leading logarithmic corrections at small transverse momentum in hadronic collisions*

Daniel de Florian and **Massimiliano Grazzini**
Institute of Theoretical Physics, ETH-Hönggerberg, CH-8093 Zürich, Switzerland

Abstract

We study the region of small transverse momenta in $q\bar{q}$- and gg-initiated processes with no colored particle detected in the final state. We present the universal expression of the $\mathcal{O}(\alpha_s^2)$ logarithmically-enhanced contributions up to next-to-next-to-leading logarithmic accuracy. From there we extract the coefficients that allow the resummation of the large logarithmic contributions. We find that the coefficient known in the literature as $B^{(2)}$ is process dependent, since it receives a hard contamination from the one loop correction to the leading order subprocess. We present the general result of $B^{(2)}$ for both quark and gluon channels. In particular, in the case of Higgs production, this result will be relevant to improve the matching between resummed predictions and fixed order calculations.

PACS: 12.38.Bx, 12.38.Cy

*Work supported in part by the EU Fourth Framework Programme 'Training and Mobility of Researchers', Network 'Quantum Chromodynamics and the Deep Structure of Elementary Particles', contract FMRX-CT98-0194 (DG 12 - MIHT) and by the Swiss National Foundation.

The process in which a system of non strongly-interacting particles of large invariant mass Q^2 (lepton pairs, gauge boson(s), Higgs boson, and so forth) is produced in hadronic collisions is a well studied subject in perturbative QCD [1]. At transverse momenta q_T^2 of order of Q^2 the cross section can be computed by using the standard QCD-improved parton model. When q_T becomes small the simple perturbative picture is spoiled. This happens because large logarithmic corrections of the form $\log \frac{Q^2}{q_T^2}$ arise due to a non complete cancellation of soft and collinear singularities between real and virtual contributions. These large logarithmic corrections can be resummed to all orders by using the Collins-Soper-Sterman (CSS) formalism [2].

We consider the class of inclusive hard scattering processes

$$h_1 h_2 \to A_1 + A_2 \ldots A_n + X \tag{1}$$

where the collision of the hadrons h_1 and h_2 produces a system of non strongly-interacting final state particles $A_1 \ldots A_n$ carrying total momentum Q and total transverse momentum q_T. According to the CSS formula, and neglecting terms which are finite in the limit $q_T \to 0$, the cross-section can be written as [1] :

$$\begin{aligned}\frac{d\sigma}{dq_T^2 dQ^2 d\phi} = \sum_{a,b,c} \int_0^1 dx_1 \int_0^1 dx_2 \int_0^\infty db \frac{b}{2} J_0(bq_T) \, \frac{d\sigma_{c\bar{c}}^{(LO)}}{d\phi} \, \delta(Q^2 - x_1 x_2 s) \\ \cdot \, \left(f_{a/h_1} \otimes C_{ca} \right) \left(x_1, \frac{b_0^2}{b^2} \right) \left(f_{b/h_2} \otimes C_{\bar{c}b} \right) \left(x_2, \frac{b_0^2}{b^2} \right) S_c(Q,b) \, ,\end{aligned} \tag{2}$$

where $d\phi = dPS(Q \to q_1, q_2, \ldots q_n)$ represents the phase space of the system of non-colored particles, $b_0 = 2e^{-\gamma_e}$ and $\sigma_{c\bar{c}}^{(LO)}$ is the leading-order cross-section (i.e., with no final state partons and therefore $q_T = 0$) for the given process ($c, \bar{c}$ can be either $q_f, \bar{q}_{f'}$ or g, g). The function C_{ab} in Eq. (2) is a process-dependent coefficient function, $J_0(bq_T)$ is the Bessel function of first kind and $f_{i/h}$ corresponds to the distribution of a parton i in a hadron h. The large logarithmic corrections are exponentiated in the Sudakov form factor

$$S_c(Q,b) = \exp \left\{ - \int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2} \left[A_c(\alpha_s(q^2)) \, \ln \frac{Q^2}{q^2} + B_c(\alpha_s(q^2)) \right] \right\} \, . \tag{3}$$

The functions A_c, B_c and C_{ab} in Eqs. (2,3) have perturbative expansions in α_s,

$$A_c(\alpha_s) = \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{2\pi} \right)^n A_c^{(n)} \quad , \tag{4}$$

$$B_c(\alpha_s) = \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{2\pi} \right)^n B_c^{(n)} \quad , \tag{5}$$

$$C_{ab}(\alpha_s, z) = \delta_{ab} \, \delta(1-z) + \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{2\pi} \right)^n C_{ab}^{(n)}(z) \quad . \tag{6}$$

[1] It is assumed that all other dimensionful invariants are of the same order Q^2.

In order to obtain the coefficients in Eqs.(4-5) at a given order, the differential cross-section at small q_T has to be computed at the same order. A comparison with the power expansion in α_s of the resummed result in Eq. (2) allows to extract the coefficients that control the resummation of the large logarithmic terms.

In this letter we study the behaviour of cross-sections at small transverse momenta at second order in α_s both in the quark and gluon channels. We find that the analytic form of the logarithmically-enhanced contributions can be computed perturbatively in a universal manner by using the recent knowledge on the infrared behaviour of tree-level [3,4] and one-loop [5] QCD amplitudes. In this way, we are able to extract the coefficients $A_c^{(1)}$, $B_c^{(1)}$, $C_{ab}^{(1)}$, $A_c^{(2)}$ and $B_c^{(2)}$ for *any* $q\bar{q}$ or gg initiated process in the class (1). Details on our calculation will be given elsewhere [6]. Here we only present and discuss our main results.

By following Ref. [7] we multiply the differential cross-section, calculated at parton level, by q_T^2 and take moments with respect to $z = Q^2/s$ defining the dimensionless quantity:

$$\Sigma(N) = \int dz\, z^N \frac{q_T^2 Q^2}{d\sigma_0/d\phi} \frac{d\sigma}{dq_T^2 dQ^2 d\phi}\,. \tag{7}$$

In the quark channel, for the sake of simplicity and in order to compare our result for $\Sigma(N)$ to the one originally obtained for Drell-Yan in Ref. [7], we restrict our attention to the *non-singlet* contribution to the cross-section defined by

$$\sigma^{NS} = \sum_{ff'} \left(\sigma_{q_f \bar{q}_{f'}} - \sigma_{q_f q_{f'}} \right)\,. \tag{8}$$

To have $q_T \neq 0$ at least one gluon must be emitted, thus $\Sigma(N)$ has the expansion:

$$\Sigma(N) = \frac{\alpha_s}{2\pi}\, \Sigma^{(1)}(N) + \left(\frac{\alpha_s}{2\pi}\right)^2 \Sigma^{(2)}(N) + \cdots \tag{9}$$

In the following we will systematically neglect in $\Sigma(N)$ all contributions that vanish as $q_T \to 0$.

In order to compute the small q_T behaviour of $\Sigma(N)$ our strategy is as follows. The singular behaviour at small q_T is dictated by the infrared (soft and collinear) structure of the relevant QCD matrix elements. At $\mathcal{O}(\alpha_s)$ this structure has been known for long time [3]. Recently, the universal functions that control the soft and collinear singularities of tree-level and one-loop QCD amplitudes at $\mathcal{O}(\alpha_s^2)$ have been computed [4,5].

By using this knowledge, and exploiting the simple kinematics of the leading order subprocess, we were able to construct *improved* factorization formulae that allow to control *all* infrared singular regions avoiding any problem of double counting [6]. We have used these improved formulae to approximate the relevant matrix elements and compute the small q_T behaviour of $\Sigma(N)$ in a completely universal manner.

The calculation at $\mathcal{O}(\alpha_s)$ is straightforward and we recover the well-known results:

$$\Sigma_{q\bar{q}}^{(1)}(N) = 2C_F \log\frac{Q^2}{q_T^2} - 3C_F + 2\gamma_{qq}^{(1)}(N) \tag{10}$$

and

$$\Sigma_{gg}^{(1)}(N) = 2C_A \log \frac{Q^2}{q_T^2} - 2\beta_0 + 2\gamma_{gg}^{(1)}(N)\,. \tag{11}$$

Here $C_F = \frac{N_c^2-1}{2N_c}$, $C_A = N_c$ and $T_R = 1/2$ are the $SU(N_c)$ QCD colour factors, $\beta_0 = \frac{11}{6}C_A - \frac{2}{3}n_f T_R$ and $\gamma_{qq}^{(1)}(N)$, $\gamma_{gg}^{(1)}(N)$ are the quark and gluon one-loop anomalous dimensions, respectively. From Eqs. (10,11) one obtains:

$$A_a^{(1)} = 2C_a \qquad\qquad B_a^{(1)} = -2\gamma_a \qquad\qquad a = q, g \tag{12}$$

where C_a and γ_a are the coefficients of the leading $(1-z)^{-1}$ singularity and $\delta(1-z)$ term in the one-loop Altarelli-Parisi kernels P_{aa}, respectively,

$$C_q = C_F \qquad\qquad C_g = C_A \qquad\qquad \gamma_q = \frac{3}{2}C_F \qquad\qquad \gamma_g = \beta_0\,. \tag{13}$$

At this order it is possible to obtain also the coefficient $C_{ab}^{(1)}$ by considering the q_T integrated distribution and including the renormalized virtual correction to the LO amplitude $c\bar{c} \to A_1 + A_2 \ldots A_n$, summed over spins and colours, which, at $\mathcal{O}(\epsilon^0)$, can be written as[2]

$$\mathcal{M}_{c\bar{c}}^{(0)\dagger}(\phi)\mathcal{M}_{c\bar{c}}^{(1)}(\phi) + \text{c.c.} = \frac{\alpha_s}{2\pi}\left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(-\frac{2C_c}{\epsilon^2} - \frac{2\gamma_c}{\epsilon} + \mathcal{A}_c(\phi)\right)|\mathcal{M}_{c\bar{c}}^{(0)}(\phi)|^2. \tag{14}$$

In Eq. (14) the structure of the poles in $\epsilon = (4-d)/2$ is universal [8] and fixed by the flavour of the incoming partons. The *finite* part $\mathcal{A}$ (which can depend on the kinematics of the final state non-colored particles) depends instead on the particular process in the class (1) we want to consider. In the case of Drell-Yan we have [9]:

$$\mathcal{A}_q^{DY} = C_F\left(-8 + \frac{2}{3}\pi^2\right)\,, \tag{15}$$

whereas for Higgs production in the $m_{top} \to \infty$ limit the finite contribution is [10]:

$$\mathcal{A}_g^H = 5C_A + \frac{2}{3}C_A\pi^2 - 3C_F \equiv 11 + 2\pi^2\,. \tag{16}$$

By using the information in Eq. (14) we obtain for $C_{ab}^{(1)}$

$$C_{ab}^{(1)}(z) = -\hat{P}_{ab}^{\epsilon}(z) + \delta_{ab}\,\delta(1-z)\left(C_a\frac{\pi^2}{6} + \frac{1}{2}\mathcal{A}_a(\phi)\right) \tag{17}$$

where $\hat{P}_{ab}^{\epsilon}(z)$ is the $\mathcal{O}(\epsilon)$ term in the Altarelli-Parisi $\hat{P}_{ab}(z,\epsilon)$ splitting kernel, given by:

[2] All our results are obtained using the factorization and renormalization prescriptions of the $\overline{MS}$ scheme and within the framework of conventional dimensional regularization.

$$\begin{aligned}
\hat{P}^{\epsilon}_{qq}(z) &= -C_F\,(1-z) \\
\hat{P}^{\epsilon}_{gq}(z) &= -C_F\,z \\
\hat{P}^{\epsilon}_{qg}(z) &= -2T_R\,z(1-z) \\
\hat{P}^{\epsilon}_{gg}(z) &= 0\,.
\end{aligned} \tag{18}$$

At order α_s the coefficients $A_a^{(1)}$ and $B_a^{(1)}$ are fully determined by the *universal* Altarelli-Parisi splitting functions. The function $C_{ab}^{(1)}$ depends instead on the process through the one-loop corrections to the *LO* matrix element. The general expression in Eq. (17) reproduces correctly the coefficient $C_{ab}^{(1)}$ for Drell-Yan [7], Higgs production in the $m_{top}\to\infty$ limit [11] and $\gamma\gamma$ production [12][3].

At second order in α_s, two different contributions to $\Sigma^{(2)}(N)$ have to be considered: the real correction corresponding to the emission of one extra parton (i.e., two gluons or a $q\bar{q}$ pair) with respect to the $\mathcal{O}(\alpha_s)$ contribution, and its corresponding virtual correction.

The double-real emission contribution is the most difficult to compute. One has to integrate over the phase space of the two unresolved final state partons keeping q_T fixed and finally perform the z integration in Eq. (7). We find that, likewise $\Sigma^{(1)}(N)$, this contribution to $\Sigma^{(2)}(N)$ is *process independent*, i.e., it does not depend on the particular process in the class (1) we want to consider.

The virtual contribution is simpler to compute and we find it to be *process dependent*. More importantly, its process dependence is fully determined by the function $\mathcal{A}$ appearing in the one-loop correction to the *LO* subprocess (see Eq. (14)).

In the following, for the sake of simplicity, we present the total results for $\Sigma^{(2)}(N)$ corresponding to the choice of the factorization and renormalization scales fixed to Q^2. Since we are interested in extracting the coefficients $A_{q,g}^{(2)}$ and $B_{q,g}^{(2)}$, as in the $\mathcal{O}(\alpha_s)$ case we concentrate on the *diagonal* $q\bar{q}$ and gg contributions to $\Sigma^{(2)}(N)$.

In the quark (non-singlet) channel we obtain:

$$\begin{aligned}
\Sigma^{(2)}_{q\bar{q}}(N) = \log^3\frac{Q^2}{q_T^2}&\left[-2C_F^2\right] \\
&+\log^2\frac{Q^2}{q_T^2}\left[9C_F^2+2C_F\beta_0-6C_F\gamma^{(1)}_{qq}(N)\right] \\
&+\log\frac{Q^2}{q_T^2}\left[C_F^2\left(\frac{2}{3}\pi^2-7\right)+C_FC_A\left(\frac{35}{18}-\frac{\pi^2}{3}\right)-\frac{2}{9}C_Fn_fT_R+2C_F\mathcal{A}_q(\phi)\right. \\
&\qquad\left.+\left(2\beta_0+12C_F\right)\gamma^{(1)}_{qq}(N)-4\left(\gamma^{(1)}_{qq}(N)\right)^2+4C_F^2\left(\frac{1}{(N+1)(N+2)}-\frac{1}{2}\right)\right] \\
&+\left[C_F^2\left(-\frac{15}{4}-4\zeta(3)\right)+C_FC_A\left(-\frac{13}{4}-\frac{11}{18}\pi^2+6\zeta(3)\right)-3C_F\mathcal{A}_q(\phi)\right. \\
&\quad+C_Fn_fT_R\left(1+\frac{2}{9}\pi^2\right)+2\gamma^{(2)}_{(-)}(N)+2C_F\gamma^{(1)}_{qq}(N)\left(\frac{\pi^2}{3}+2\frac{1}{(N+1)(N+2)}\right)
\end{aligned}$$

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[3] The actual expression of the coefficient $C_{qq}^{(1)}$ for ZZ production reported in Ref. [13] is not fully correct.

$$+2\gamma_{qq}^{(1)}(N)\mathcal{A}_q(\phi) - 2C_F(\beta_0 + 3C_F)\left(\frac{1}{(N+1)(N+2)} - \frac{1}{2}\right)\Bigg] \,, \tag{19}$$

whereas for the gluon channel the result is:

$$\begin{aligned}
\Sigma_{gg}^{(2)}(N) = {}& \log^3\frac{Q^2}{q_T^2}\left[-2C_A^2\right] \\
&+ \log^2\frac{Q^2}{q_T^2}\left[8C_A\beta_0 - 6C_A\gamma_{gg}^{(1)}(N)\right] \\
&+ \log\frac{Q^2}{q_T^2}\Bigg[C_A^2\left(\frac{67}{9} + \frac{\pi^2}{3}\right) - \frac{20}{9}C_A n_f T_R + 2C_A\mathcal{A}_g(\phi) \\
&\qquad +2\beta_0\left(\gamma_{gg}^{(1)}(N) - \beta_0\right) - 4\left(\gamma_{gg}^{(1)}(N) - \beta_0\right)^2 - 4n_f\,\gamma_{gq}^{(1)}(N)\gamma_{qg}^{(1)}(N)\Bigg] \\
&+ \Bigg[C_A^2\left(-\frac{16}{3} + 2\zeta(3)\right) + 2C_F n_f T_R + \frac{8}{3}C_A n_f T_R - 2\beta_0\left(\mathcal{A}_g(\phi) + C_A\frac{\pi^2}{6}\right) \\
&\quad +2\gamma_{gg}^{(2)}(N) + 2\gamma_{gg}^{(1)}(N)\left(\mathcal{A}_g(\phi) + C_A\frac{\pi^2}{3}\right) + 4C_F n_f\,\gamma_{qg}^{(1)}(N)\frac{1}{(N+2)}\Bigg] \,.
\end{aligned} \tag{20}$$

In Eq. (19) $\gamma_{(-)}^{(2)}(N)$ is the non singlet space-like two-loop anomalous dimension [15], in Eq. (20) $\gamma_{gg}^{(2)}(N)$ is the singlet space-like two-loop anomalous dimension [16] , $\zeta(n)$ is the Riemann ζ function ($\zeta(3) = 1.202...$) and the function $\mathcal{A}_a(\phi)$ is defined through Eq. (14). The coefficients $\frac{1}{(N+1)(N+2)}$ and $\frac{1}{(N+2)}$ have origin on the N moments of $-\hat{P}_{qg}^{\epsilon}(z)$ and $-\hat{P}_{gq}^{\epsilon}(z)$, respectively.

The N-dependent part of the results in Eqs. (19,20) agrees[4] with the one obtained from the second order expansion of Eq. (2) (see e.g. Ref. [14]). By comparing also the N-independent part we obtain for $A^{(2)}$:

$$A_a^{(2)} = K A_a^{(1)} \qquad\qquad K = C_A\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - n_f T_R\frac{10}{9} \tag{21}$$

in agreement with the results of Ref. [17,18]. Moreover we find that $B^{(2)}$ can be expressed as:

$$B_a^{(2)} = -2\,\delta P_{aa}^{(2)} + \beta_0\left(\frac{2}{3}C_a\pi^2 + \mathcal{A}_a(\phi)\right) \qquad\qquad a = q, g \tag{22}$$

where $\delta P_{aa}^{(2)}$ are the coefficients of the $\delta(1-z)$ term in the two-loop splitting functions $P_{aa}^{(2)}(z)$ [15,16], and are given by

$$\begin{aligned}
\delta P_{qq}^{(2)} &= C_F^2\left(\frac{3}{8} - \frac{\pi^2}{2} + 6\zeta(3)\right) + C_F C_A\left(\frac{17}{24} + \frac{11\pi^2}{18} - 3\zeta(3)\right) - C_F n_f T_R\left(\frac{1}{6} + \frac{2\pi^2}{9}\right) \\
\delta P_{gg}^{(2)} &= C_A^2\left(\frac{8}{3} + 3\zeta(3)\right) - C_F n_f T_R - \frac{4}{3}C_A n_f T_R \,.
\end{aligned} \tag{23}$$

[4]This can also be regarded as an independent re-evaluation of the two-loop anomalous dimensions.

From Eq. (22) we see that $B^{(2)}$, besides the $-2\,\delta P_{aa}^{(2)}$ term which matches the expectation from the $\mathcal{O}(\alpha_s)$ result, receives a *process-dependent* contribution controlled by the one-loop correction to the LO amplitude (see Eq. (14)). We conclude that the Sudakov form factor in Eq. (3) is actually process dependent beyond next-to-leading logarithmic accuracy. The interpretation of this result will be given elsewhere [19].

However, by using the general expression in Eq. (22) it is possible to obtain $B^{(2)}$ for a given process just by computing the one-loop correction to the LO amplitude for that process. For the case of Drell-Yan, by using Eq. (15), our result for $\Sigma_{q\bar{q}}^{(2)}(N)$ agrees with the one of Ref. [7], and we confirm:

$$B_q^{(2)DY} = C_F^2\left(\pi^2 - \frac{3}{4} - 12\zeta(3)\right) + C_F\,C_A\left(\frac{11}{9}\pi^2 - \frac{193}{12} + 6\zeta(3)\right) + C_F\,n_f\,T_R\left(\frac{17}{3} - \frac{4}{9}\pi^2\right). \tag{24}$$

In the case of Higgs production in the $m_{top} \to \infty$ limit, by using Eq. (16) we find:

$$B_g^{(2)H} = C_A^2\left(\frac{23}{6} + \frac{22}{9}\pi^2 - 6\zeta(3)\right) + 4C_F\,n_f\,T_R - C_A\,n_f\,T_R\left(\frac{2}{3} + \frac{8}{9}\pi^2\right) - \frac{11}{2}C_F\,C_A\,. \tag{25}$$

In particular, this result allows to improve the present accuracy of the matching between resummed predictions [20] and fixed order calculations [21].

Summarizing, we have studied the logarithmically-enhanced contributions at small transverse momentum in hadronic collisions at second order in perturbative QCD. The calculation was performed in an process-independent manner, allowing us to show that the Sudakov form factor is actually process dependent beyond next-to-leading logarithmic accuracy. We have provided a general expression for the coefficient $B^{(2)}$ for both quark and gluon initiated processes.

We would like to thank Stefano Catani and Zoltan Kunszt for many valuable discussions and comments, Christine Davies for providing us with a copy of her Ph.D. thesis and Luca Trentadue, James Stirling and Werner Vogelsang for discussions.

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